parameter f_w'' has been obtained analytically for stagnation line, the following expression can be used to check with the values that can be obtained by using the present method:

$$f_w'' = (aC/K^2)/3 \tag{12}$$

The application of Eq (11) in finding the three-dimensional supersonic laminar boundary layer characteristics has been studied by obtaining results for a specific example for a range of circumferential angle of 90° A numerical method by using the digital computer was developed

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Normal Shock Location in Underexpanded Gas and Gas-Particle Jets

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ONE of the important parameters pertinent to supersonic jets issuing from underexpanded nozzles is the distance from the nozzle exit plane to the normal shock wave existing in such flows This note presents results of experimental determination of this distance for both gas-only and gas-particle plumes The gas-only experiments were conducted in order to determine the effect of gas specific heat ratio on the normal shock position; this was accomplished by using various gases with different specific heat ratios Plumes containing macroscopic (micron-sized) condensed-phase particles also are of interest because of their relation to metallized solid propellant rocket exhausts Thus, experiments were also performed to determine the location of the normal shock as a function of particle loading in gas-particle jets, and it was found that the normal shock moves closer to the nozzle exit plane as particle loading is increased An empirical correlation of the data is presented which is valid for both the gas only and the gas-particle plumes

The test facility is shown in Fig 1 The system consists of a high-pressure (stagnation pressure approximately equal to 100 psia) gas-only or gas-particle mixture which passes through a nozzle into a low pressure reservoir of 3800 ft^3 volume The mixing of the particles and the gas is accomplished by forcing the particles into the gas stream by means of a motor-driven piston In addition to schlieren photography of the plume, instrumentation includes measurements of stagnation, exit plane, and tank pressures and gas and particle flow rates Two different conical nozzles were used Each has a throat diameter of 0 260 in and an ex pansion cone half-angle of 15°, but one has an area ratio (exit to throat) of 1 385 whereas the other is 3 868

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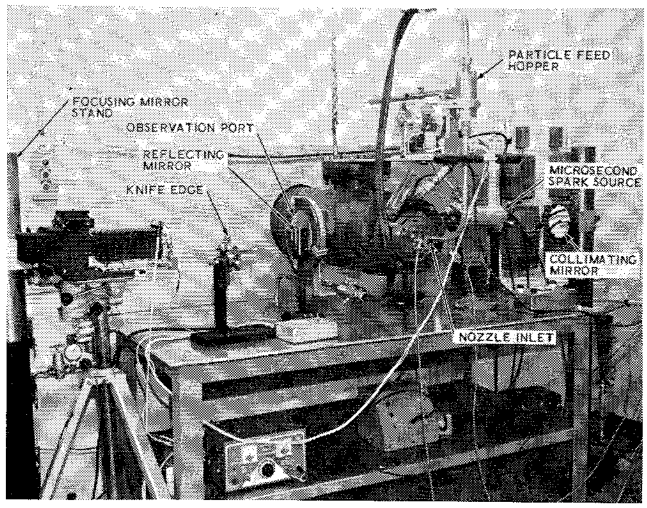

Fig 1 Gas-particle flow facility

The flow parameters that were varied consist of the particle-to-gas mass fraction ϕ, the ratio of the exit plane static pressure to the ambient (tank) pressure p /p_∞, the nozzle exit Mach number M , and the ratio of specific heats γ The distance to the normal shock was determined directly from the schlieren photographs Gas-only experiments using nitrogen, carbon dioxide, and helium were made to provide a basis for comparison with the data of the gas-particle runs as well as to determine the effect of γ on the distance to the normal shock It was found that the data from these tests for both nozzles and all three gases could be represented by

$$\frac{x_g}{d} = 0\,69M \left(\frac{\gamma p}{p_\infty}\right)^{1/2} \tag{1}$$

where x_g is the distance to the normal shock for gas-only flows, d is the nozzle exit diameter, and M is the nozzle exit Mach number Some of the experimental data are shown compared with Eq (1) in Fig 2 Equation (1) was also compared with experimental results of several other investigators[1–5] covering a range of γ from 1 22 to 1 67, a range of M from 1 to 3 and a range of p /p_∞ from 1 to about 550 Included were 5 different gases for the cold flow tests and fir-

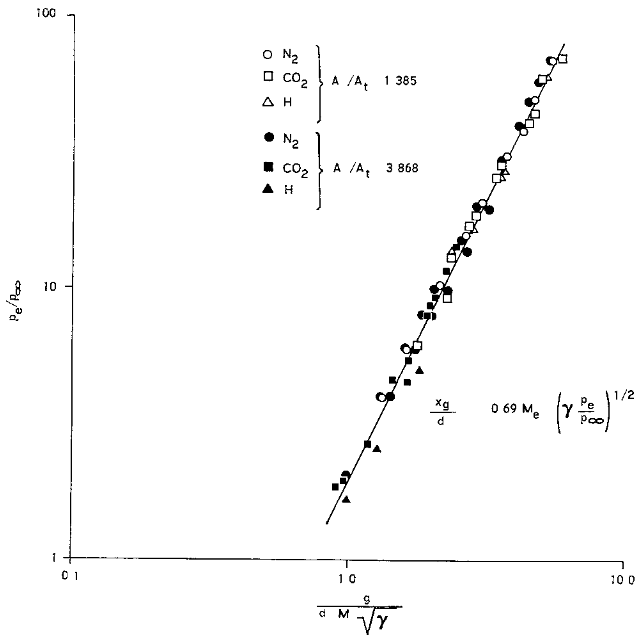

Fig 2 Distance to the normal shock in gas-only flows normalized by d M $\gamma^{1/2}$

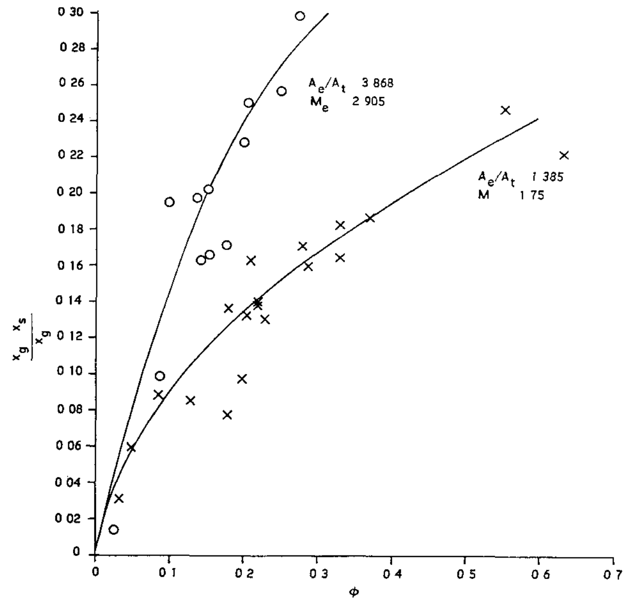

Fig 3 Percentage change in distance to the noimal shock as a function of particle mass fraction

ings of liquid propellant rocket engines About 90% of the data points fell in a region within ±10% of values predicted by Eq (1) The greatest deviation occurred at an exit Mach number of unity, principally in the data of Bier and Schmidt,[3] which varied as much as 31% higher than values predicted by Eq (1) However, other Mach one data[1 2] agree well It is not surprising that variation between different data exists, since a variety of experimental techniques were used to determine the location of the normal shock, ranging from shadowgraph and different forms of schlieren to radiancy measurements in some hot firings Also, because the normal shock becomes somewhat concave at Mach numbers approaching one, and at high values of p/p_∞, it is possible that different interpretations of what is termed the distance to the normal shock were used, depending on the experimental techniques used to find the shock location

All gas-particle tests utilized one batch of alumina particles The size distribution was obtained by counting particles appearing on electron micrographs of several random samples; the number-mean diameter was found to be 2 8 μ and the mass-mean diameter was found to be 9 3 μ

Some results of experiments using the alumina particles suspended in nitrogen gas are shown in Fig 3 (the exit Mach numbers shown there refer to gas-only values) During the course of the experiments, it was determined that the percentage change in position of the normal shock with the addition of particles was independent of pressure ratio This means that at a fixed value of ϕ, the distance to the normal shock should have the same functional relationship with respect to p/p_∞ as would a gas-only plume, but displaced by some function $f(\phi)$ An empirical fit to the experimental data yielded

$$f(\phi) = \frac{1}{1 + 0\ 197M^{1\ 45}\phi^{0\ 65}} \quad (2)$$

so that, for gas-particle flows, the distance to the normal shock x_s can be expressed by

$$\frac{x_s}{d} = \frac{0\ 69M_e}{1 + 0\ 197M^{1\ 45}\phi^{0\ 65}}\left(\frac{\gamma p}{p_\infty}\right)^{1/2} \quad (3)$$

The greatest deviation of experimental data from Eq (3) was 5%

These experiments and the resulting Eq (3) predict a significant reduction in the distance to the normal shock in metallized solid propellant exhausts Observations by Sutton[6] show that this is indeed the case; in fact, Sutton's observations are in good agreement with Eq (3)

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Effect of Gage Material on Convective Heat Transfer

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RECENTLY[1 2] there has been some discussion about the effect of gage materials on convective heat-transfer measurements in shock tubes In order to check the effect of gage materials, a series of convective heat-transfer tests have been conducted in a 9% CO_2, 1% A, and 90% N_2 mixture

Nickel and gold calorimeter data were obtained and compared with previously obtained platinum data in the same mixture[3] The property values for gold and nickel were taken from the *Handbook of Chemistry and Physics* The properties of platinum were guaranteed by the manufacturer for high-purity platinum The values used in the calculation of the convective heat transfer are shown in Table 1

The resistance was separately determined for each run The platinum data was corrected for the temperature variation of the properties[4] This correction is of the order of 10%

The models were hemisphere-cylinders 1 in in diameter All tests were conducted in the Jet Propulsion Laboratory

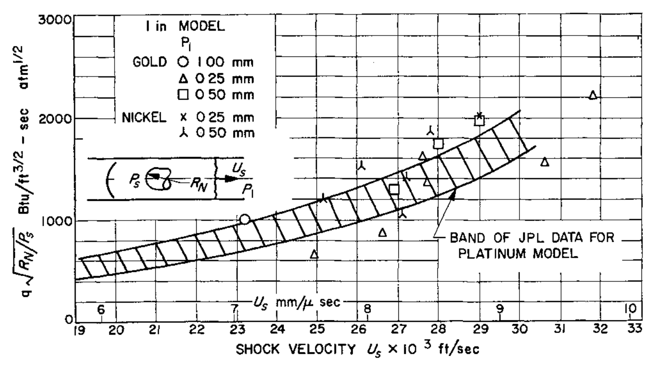

Fig 1 Convective heat transfer in 90% N_2-9% CO_2-1% A effect of different gage materials

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